Efficient Iterative Source-Channel Decoding Using Irregular Index Assignments

Laurent Schmalen[1], Peter Vary[1], Thorsten Clevorn[2], Birgit Schotsch[1]

[1] Institute of Communication Systems and Data Processing (ind)
RWTH Aachen University, Germany, {schmalen|vary|schotsch}@ind.rwth-aachen.de

[2] Infineon Technologies, COM Development Center NRW
Duisburg, Germany, thorsten.clevorn@infineon.com

Abstract

We present a system for iterative source-channel decoding (ISCD) using irregular index assignments: The concept of irregular codes is applied to the index assignment of a scalar quantizer. The optimization performed in the EXIT chart enables near optimum transmission. The irregular index assignments are constructed by using high-rate block codes. This construction allows to use a very simple stopping criterion at the receiver and thereby to potentially reduce the number of iterations. We demonstrate the performance of this system by means of a simulation example over an AWGN channel with hard decision at the output. Furthermore we present simple yet effective measures for the case that no channel state information (e.g., instantaneous bit error rate) is available.

1 Introduction

With the discovery of Turbo codes, channel coding close to the Shannon limit became possible with moderate computational complexity. In the past years, the Turbo principle of exchanging *extrinsic* information between separate channel decoders has also been adapted to other receiver components.

To exploit the residual redundancy in source coded parameters such as scale factors or predictor coefficients for speech, audio, and video signals in a Turbo process, *iterative source-channel decoding* (ISCD) has been presented in [1], [2] as a means to further improve the quality of *soft decision source decoding* (SDSD) [3], [4]. This residual redundancy occurs due to non-ideal source encoding due to, e.g., delay or complexity constraints.

Most previous publications on ISCD have been focusing on the AWGN channel with perfect soft information available at the receiver. However, in some transmission scenarios it might not be possible to transfer soft information from the physical layer to an upper layer where source-channel decoding may take place. For this reason, we consider a transmission over an AWGN channel with binary quantization of the received values. This channel can be considered as a *binary symmetric channel* (BSC).

It is known that the inner channel code of a capacity-achieving serially concatenated system should be of rate $r = 1$ [5]. If this inner channel code is fixed, the outer code can be matched quite well to the inner rate-1 code using the principles of irregular codes [6], [7]. Irregular codes allow a simple optimization of the outer component by making use of EXIT charts [8]. In this contribution, we employ the concept of irregular codes to be used as (redundant) index assignment, i.e., the assignment of bit patterns to codebook indices, of a (scalar) quantizer to get so-called *irregular index assignments* (IIA), which extend the concept of redundant index assignments [9], [10], [11]. Additionally, the utilized IIA design guidelines permit to implement a very simple stopping criterion at the receiver, limiting the necessary amount of iterations performed in the case of good channel conditions. Such a stopping criterion is extremely important in mobile applications where the reduction of the power consumption is one of the main optimization targets.

Often, no *channel state information* (CSI), such as the instantaneous bit error rate or the channel signal-to-noise ratio, is available at the receiver. Without CSI, the *maximum a posteriori* (MAP) algorithm, often employed as channel decoder in ISCD systems, is not able to successfully decode, even in good channel conditions. By applying several measures to compensate for the unknown CSI, we show that a performance can be achieved which is comparable to that of the corresponding system with full CSI.

The paper is structured as follows: In Section 2 we give an overview of the system and present the different modules used. In Section 3, the concept of Irregular Index Assignments is introduced and explained while the generation guideline of the index assignments is used to derive a fairly straightforward stopping criterion in Section 4. In Section 5 we give modification guidelines for the case that no channel state information is available at the receiver. The paper concludes with a simulation example in Section 6 showing the near optimum performance of the proposed system.

2 System Model

In what follows, we will give a brief review of the *iterative source-channel decoding* (ISCD) system. In Fig. 1 the baseband model of ISCD is depicted. At time instant τ a source encoder generates a frame $\underline{u}_\tau = (u_{1,\tau}, \dots u_{K_S,\tau})$ of K_S unquantized source codec parameters $u_{\kappa,\tau}$, with $\kappa \in \{1, \dots, K_S\}$ denoting the position in the frame. The single elements

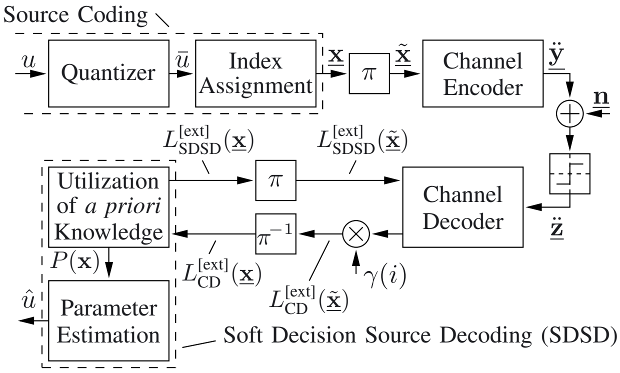

Fig. 1. Baseband model of the utilized ISCD system (simplified notation, e.g., u instead of $u_{\kappa,\tau}$)

$u_{\kappa,\tau}$ of $\underline{u}_\tau$ are assumed to be statistically independent from each other. Each value $u_{\kappa,\tau}$ is individually mapped to a quantizer reproduction level $\bar{u}_{\kappa,\tau}$, with $\bar{u}_{\kappa,\tau} \in \mathbb{U}_\kappa = \{\bar{u}_{\kappa,\tau}^{(0)}, \ldots, \bar{u}_{\kappa,\tau}^{(Q_\kappa-1)}\}$. The set $\mathbb{U}$ denotes the quantizer codebook with a total number of $|\mathbb{U}_\kappa| = Q_\kappa$ codebook entries. The number of quantizer levels is assumed to be $Q_\kappa = 2^{M_\kappa}$. A unique bit pattern $\mathbf{x}_{\kappa,\tau}$ of M_κ^* bits (with $M_\kappa^* > M_\kappa$) is assigned to each quantizer level $\bar{u}_{\kappa,\tau}$ selected at time instant τ according to the index assignment

$$\begin{aligned}\Gamma_\kappa: \quad & \mathbb{U}_\kappa \to \mathbb{F}_2^{M_\kappa^*}\\ & \bar{u}_{\kappa,\tau} \mapsto \mathbf{x}_{\kappa,\tau}\end{aligned}$$

with $\mathbb{F}_2 = \{0, 1\}$. In the following, we assume that all codec parameters are quantized using the same codebook, i.e., $\mathbb{U}_\kappa = \mathbb{U}$ and $M_\kappa = M, \forall \kappa \in \{1, \ldots, K_S\}$. Although the number of quantization levels is assumed to be identical for all parameters, the index assignment can differ from parameter to parameter. For notational convenience we omit the time index τ in the following.

The single bits of a bit pattern $\mathbf{x}_\kappa$ are indicated by $x_\kappa^{(m)}$, $m \in \{1, \ldots, M_\kappa^*\}$. If $M_\kappa^* > M = M_\kappa$, the index assignment Γ_κ introduces redundancy and can then be considered to be the composite function $\Gamma_\kappa = \zeta_\kappa \circ \breve{\Gamma}_{\text{NB}}$ (i.e., $\Gamma_\kappa(\bar{u}) = (\zeta_\kappa \circ \breve{\Gamma}_{\text{NB}})(\bar{u}) = \zeta_\kappa(\breve{\Gamma}_{\text{NB}}(\bar{u}))$) with

$$\begin{aligned}\breve{\Gamma}_{\text{NB}}: \mathbb{U} &\to \mathbb{F}_2^M & \text{and} \qquad \zeta_\kappa: \mathbb{F}_2^M &\to \mathbb{F}_2^{M_\kappa^*}\\ \bar{u} &\mapsto \breve{\mathbf{x}} & \breve{\mathbf{x}} &\mapsto \mathbf{x}_\kappa\,.\end{aligned}$$

The function $\breve{\Gamma}_{\text{NB}}$ performs a *natural binary* index assignment, i.e., the binary representation of the codebook index of $\bar{u}$ is assigned to $\breve{\mathbf{x}}$. The function ζ_κ can be regarded as being a (potentially non-linear) block code of rate $r_\kappa^{\mathsf{IA}} = M/M_\kappa^*$. The concept of non-linear block codes employed as redundant index assignments has been successfully utilized in, e.g., [12], [11]. In this paper however, we only consider linear block codes and refer to Section 3 for a detailed description. After the index assignment, K_S bit patterns are grouped to a frame of bit patterns $\underline{\mathbf{x}} = (\mathbf{x}_1, \ldots, \mathbf{x}_{K_S})$ consisting of $\sum_{\kappa'=1}^{K_S} M_{\kappa'}^* = K_S \cdot \bar{M}^*$ bits. The overall rate of the index assignment is thus

$$r^{\mathsf{IA}} = \frac{K_S \cdot M}{\sum\limits_{\kappa'=1}^{K_S} M_{\kappa'}^*} = \frac{M}{\bar{M}^*}\,, \tag{1}$$

with $\bar{M}^*$ the average number of bits per parameter. The frame $\underline{\mathbf{x}}$ of bits is re-arranged by a bit interleaver π in a deterministic, pseudo-random like manner. The interleaved frame with $K_S \cdot \bar{M}^*$ bits is denoted as $\underline{\tilde{\mathbf{x}}}$.

For channel encoding of a frame $\underline{\tilde{\mathbf{x}}}$, we use a recursive convolutional code of constraint length $J+1$ and of rate r^{C}. In this paper, we restrict ourselves to rate $r^{\mathsf{C}} = 1$ recursive, non-systematic convolutional codes. The encoded frame is denoted by $\underline{\mathbf{y}}$. The bits y_k of $\underline{\mathbf{y}}$ are indexed by $k \in \{1, \ldots, K_S \cdot \bar{M}^* + J\}$. Prior to transmission over the channel, the encoded bits y_k are mapped to bipolar bits $\ddot{y}_k$ forming a sequence $\underline{\ddot{\mathbf{y}}} \in \{\pm 1\}^{K_S \cdot \bar{M}^* + J}$. We only consider BPSK modulation in this paper in order to demonstrate the concept, which can easily be extended to include higher order modulation schemes [13] or channel equalization [14]. Note that in Fig. 1 the baseband model is considered.

On the channel, the modulation symbols $\ddot{y}_k$ (with symbol energy $E_{\text{s}} = 1$) are subject to additive white Gaussian noise (AWGN) with known power spectral density $\sigma_n^2 = N_0/2$. After transmission, a hard-decision is performed on the received symbols z_k, i.e., $\ddot{z}_k = \text{sign}\{z_k\}$. This implies that the channel can be modelled as a *binary symmetric channel* (BSC) with bit error probability

$$P_b = P\left(\ddot{z}_k \neq \ddot{y}_k\right) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_{\text{s}}}{N_0}}\right)\,, \tag{2}$$

with erfc denoting the complementary error function.

The received symbols $\ddot{z}_k \in \{\pm 1\}$ are transformed to L-values [15] prior to being evaluated in a Turbo process which exchanges *extrinsic* reliabilities between channel decoder (CD) and soft decision source decoder (SDSD). If *channel state information* (CSI) is available at the receiver, the L-values of the received symbols are obtained by [15]

$$L(\ddot{z}_k) = \log_{\text{e}}\left(\frac{1-P_b}{P_b}\right)\cdot \ddot{z}_k =: L_c \cdot \ddot{z}_k \tag{3}$$

and if no CSI (i.e., $\frac{E_{\text{s}}}{N_0}$ or P_b) is available at the receiver, the L-values are given by

$$L(\ddot{z}_k) = \check{L}_c \cdot \ddot{z}_k\,, \tag{4}$$

with $\check{L}_c$ being a receiver parameter. The adjustement of $\check{L}_c$ will be explained in Section 6. After channel decoding, the L-values at the decoder output can optionally be scaled by a factor $\gamma(i)$, where i denotes the iteration counter, i.e., $\gamma(i)$ is constant during one iteration.

The channel decoder used in this paper is based on the LogMAP algorithm [16], [15] or on the MaxLogMAP approximation [17]. For the equations for computing the *extrinsic* probabilities or their respective L-values of the SDSD, the reader is referred to the literature, e.g., [2], [3], [18]. Note that the redundancy of the index assignment, introduced by the function ζ_κ, is not explicitly decoded at the receiver but implicitly used to calculate better estimates of the codebook indices given the input L-values. For details, we refer the reader to [9], [11].

3 Irregular Index Assignments

According to [5], a necessary condition for a serially concatenated system to be capacity achieving is an inner component with code rate $r = 1$. For the setup introduced in Section 2, this means that the channel code should be of rate $r^{\mathsf{C}} = 1$. For a given channel code, the goal is to find a perfectly matching outer component (source code in our case) to the given rate-1 channel code. This task can be solved for example by the concept of irregular codes [7], [6]. Irregular codes, originally proposed for convolutional codes, use several component codes of different rates in one block (e.g., by changing the puncturing rule) to obtain an overall rate-r^{Outer} outer code. As the EXIT characteristic of the resulting code corresponds to the weighted sum of the component codes' characteristics (where the weights correspond to the fractions of code bits being encoded by the respective component code), an optimization algorithm can be formulated [7]. This algorithm allows to optimize the weights in order to get an (almost) perfectly matching characteristic.

We extend the concept of irregular codes to the index assignment in order to obtain *irregular index assignments* (IIA). As stated in Section 2, the index assignment for the parameter u_κ comprises a block code ζ_κ of rate $r_\kappa^{\mathsf{IA}} = M/M_\kappa^*$. Instead of using the same amount of bit redundancy $M_\kappa^* = \bar{M}^*$ for each parameter in order to achieve an overall rate $M/\bar{M}^*$ outer encoding, we use the concept of irregular codes and vary M_κ^* for each parameter. This allows us to use the optimization algorithm in [7] to optimize the index assignments and to get an SDSD EXIT characteristic which matches the channel decoder characteristic considerably well.

In the following, we present a simple design guideline in order to generate redundant index assignments with rates $r_\kappa^{\mathsf{IA}} = M/M_\kappa^*$, $M_\kappa^* \in \{M+1, \ldots, M_{\text{max}}^*\}$ needed for the optimization of the IIA. The guideline starts with an (almost) arbitrary generator matrix $\mathbf{G} = (g_{i,j})_{M \times M_{\text{max}}^*}$ of size $\dim \mathbf{G} = M \times M_{\text{max}}^*$ and with elements $g_{i,j} \in \mathbb{F}_2$. A generator matrix $\mathbf{G}_{M^*}$ for a rate $r_\kappa^{\mathsf{IA}} = M/M_\kappa^*$ index assignment is then obtained by

$$\mathbf{G}_{M^*} = \mathbf{G} \cdot \begin{pmatrix} \mathbf{I}_{M^*} \\ \mathbf{0} \end{pmatrix} \tag{5}$$

with $\mathbf{I}_{M^*}$ denoting the $M^* \times M^*$ identity matrix and $\mathbf{0}$ the $(M_{\text{max}}^* - M^*) \times M^*$ all-zero matrix. The only conditions we fix for $\mathbf{G}$ are:

a) $\mathbf{G}$ is a generator matrix for a systematic linear block code, i.e., $\mathbf{G}$ can be written as

$$\mathbf{G} = \begin{pmatrix} \mathbf{I}_M & \mathbf{P} \end{pmatrix}. \tag{6}$$

b) the block code generated by $\mathbf{G}_{M+1}$ has a minimum Hamming distance $d_{\text{Ham}}(\mathbf{G}_{M+1}) \geq 2$.

The second condition is necessary for the EXIT characteristic to reach the $(1,1)$ point [11] and is accomplished if $\mathbf{G}_{M+1}$ realizes a parity check code, i.e., $\mathbf{G}_{M+1} = (\mathbf{I}_M \;\; \mathbf{1})$.

We illustrate the generation of irregular index assignments by means of an example. $K_S = 250$ source parameters modelled by K_S independent 1st order Gauss-Markov processes with auto-correlation $\rho = 0.9$ are quantized using a $Q = |\mathbb{U}| = 16$ level Lloyd-Max codebook, i.e., $M_\kappa = M = 4$. Furthermore, we assume that the overall coding rate of the index assignment shall be of rate $r^{\mathsf{IA}} = \frac{1}{2}$, which gives an average number of $\bar{M}^* = 8$ bits per source parameter. The channel code is a memory $J = 3$, rate-1 recursive convolutional code with (octal) generator polynomials $\left(\frac{10}{17}\right)_8$. An exemplary generator matrix $\mathbf{G}$ for $M = 4$ and $M_{\text{max}}^* = 15$, fulfilling conditions a) and b) and generating redundant index assignments with rates $4/5, 4/6, \ldots, 4/15$ could be

$$\mathbf{G} = \begin{pmatrix} 1&0&0&0&1&1&1&1&0&1&1&1&0&0&0 \\ 0&1&0&0&1&1&1&0&1&0&0&1&1&1&0 \\ 0&0&1&0&1&1&0&1&1&0&1&0&1&0&1 \\ 0&0&0&1&1&0&1&1&1&1&0&0&0&1&1 \end{pmatrix}. \tag{7}$$

The index assignments generated by using the generator matrix of (7) with (5) for realizing the Block Code ζ_κ are denoted by $\text{BC}_{M^*}^{Q}$.

Example: The block code index assigment BC_6^{16} is given by $\text{BC}_6^{16} = \{\mathbf{x} | \mathbf{x} = \Gamma(\bar{u}), \bar{u} = \bar{u}^{(0)}, \ldots, \bar{u}^{(Q-1)}\} = \{0, 6, 13, 15, 23, 25, 30, 36, 43, 45, 50, 56, 60, 66, 73, 75\}$ in octal representation with the least significant bit corresponding to $x^{(M^*)}$. For instance, to the quantizer reproduction level $\bar{u}^{(5)}$, the natural binary representation $\breve{\mathbf{x}} = (0101)_2$ is assigned, leading to

$$\mathbf{x} = \breve{\mathbf{x}} \cdot \mathbf{G}_6 = (0101) \begin{pmatrix} 1&0&0&0&1&1 \\ 0&1&0&0&1&1 \\ 0&0&1&0&1&1 \\ 0&0&0&1&1&0 \end{pmatrix} = (010101)_2 = (25)_8.$$

For an overall rate-$\frac{1}{2}$ transmission with the given parameters, it can be observed that a minimum channel quality of $E_s/N_0 \approx -2.83\,\text{dB}$ is necessary to reach a reconstruction SNR of the decoded parameters of $\approx 20\,\text{dB}$. This channel quality is obtained by calculating the *optimum performance theoretically attainable* (OPTA) [19], using the capacity of the Hard-Output, Binary-Input AWGN (HO-BIAWGN) channel [20]

$$\mathcal{C}^{[\text{HO-BIAWGN}]} = \frac{1}{2}\left(\text{erfc}\,(\lambda) \log_2 \text{erfc}\,(\lambda) + \text{erfc}\,(-\lambda) \log_2 \text{erfc}\,(-\lambda)\right) \tag{8}$$

with

$$\lambda = \sqrt{\frac{E_\text{s}}{N_0}} = \left(2\sigma_n^2\right)^{-\frac{1}{2}}. \tag{9}$$

See Fig. 3 for an illustration of the OPTA limit in this case. We perform the optimization, however, using a slightly higher channel quality of $E_\text{s}/N_0 = -2.6\,\text{dB}$.

The EXIT characteristics of the channel decoder $\mathcal{T}^{\mathsf{CD}}$ and the characteristics of the different index assignments are illustrated in Fig. 2. It can be seen that the

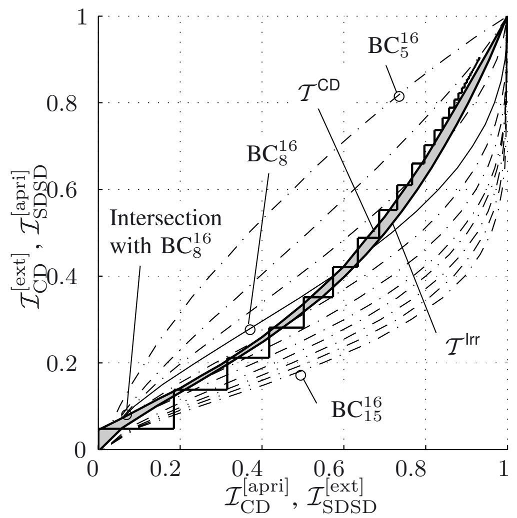

Fig. 2. EXIT chart analysis of the irregular index assignments at $E_{\mathrm{s}}/N_0 = -2.6\,\mathrm{dB}$ ($E_u/N_0 = 6.43\,\mathrm{dB}$)

trajectory of the index assignment BC$_8^{16}$, meeting the rate requirements, has an intersection with the channel decoder characteristic, resulting in a decoder failure. The optimization of the irregular index assignment leads to the characteristic $\mathcal{T}^{\mathsf{Irr}}$, matching considerably well the channel decoder characteristic with an open decoding tunnel.

The results of the optimization are summarized in Table I. The optimization determines the weights α_ℓ. The outcome of the algorithm is that not all index assignments have to be used in order to generate a good matching irregular index assignment but only five of them. The α_ℓ are the weighting factors of the EXIT characteristics and they also determine the fraction of bits to be assigned to each index assignment. From these fractions $\alpha_\ell K_S \bar{M}^*$ the corresponding $K_{S,\ell}$ (number of source parameters assigned to each index assignment) can be calculated by

$$K_{S,\ell} = \mathrm{rnd}\left[\alpha_\ell K_S \bar{M}^* \frac{r_\ell^{\mathsf{IA}}}{M}\right] = \mathrm{rnd}\left[\alpha_\ell K_S \frac{r_\ell^{\mathsf{IA}}}{r^{\mathsf{IA}}}\right], \quad (10)$$

with rnd being an appropriate rounding operation such that $\sum_{\forall i} K_{S,\ell} = K_S$. Note that the concept of irregular index assignments introduces no noteworthy additional computational complexity at the receiver, which mainly depends on the number of quantization levels per parameter (which has been fixed to $Q = 2^M$ in this contribution).

The decoding trajectory using the IIA system is also depicted in Fig. 2. It can be seen that during the first iterations, the trajectory overshoots the source decoder characteristic. This behavior has been analyzed and described in [21]. During the last iterations, the trajectory overshoots the characteristic of the channel decoder. This behavior has already been observed in [7] and is due to the relatively small bit interleaver of size 2000 bits.

TABLE I

RESULT OF THE IRREGULAR INDEX ASSIGNMENT EXAMPLE

Rate r_ℓ^{IA}	Γ_ℓ	α_ℓ	$\alpha_\ell K_S \bar{M}^*$	$K_{S,\ell}$
4/15	BC$_{15}^{16}$	0.255	510	$K_S^{(4/15)} = 34$
4/14	BC$_{14}^{16}$	0.161	322	$K_S^{(4/14)} = 23$
4/7	BC$_7^{16}$	0.189	378	$K_S^{(4/7)} = 54$
4/6	BC$_6^{16}$	0.285	570	$K_S^{(4/6)} = 95$
4/5	BC$_5^{16}$	0.110	220	$K_S^{(4/5)} = 44$
$r^{\mathsf{IA}} = \frac{1}{2}$		$\sum = 1$	$\sum = K_S \bar{M}^* = 2000$	$\sum = K_S = 250$

4 Stopping Criterion

The generation of the (irregular) index assignments using a generator matrix as presented in Section 3 enables the receiver to apply a simple yet effective stopping criterion. In good channel conditions, it is generally not necessary to perform more than only a few iterations. We use a well-known concept from *low-density parity-check* (LDPC) decoding [22], [23] and evaluate the parity check equations of the index assignment after each iteration. If all equations are fulfilled, the iterative process can be aborted.

If the generator matrix $\mathbf{G}$ is systematic according to (6), the parity check matrix $\mathbf{H}_{M^*}$ for the index assignment generated by $\mathbf{G}_{M^*}$ can easily be determined by

$$\mathbf{H}_{M^*} = \begin{pmatrix} \mathbf{P} \cdot \begin{pmatrix} \mathbf{I}_{M^*-M} \\ \mathbf{0} \end{pmatrix} \\ \mathbf{I}_{M^*-M} \end{pmatrix}^T \quad (11)$$

with $\mathbf{0}$ denoting the $(M^*_{\mathsf{max}} - M^*) \times (M^* - M)$ all-zero matrix.

Hence, a total number of $M^*_\kappa - M$ parity check equations can be evaluated for each parameter u_κ. The parity checks are performed based on the hard decisions of the extrinsic L-values $L_{\mathsf{SDSD}}^{[\mathsf{ext}]}(\underline{\mathbf{x}})$ at the output of the source decoder. If all parity checks of all parameters in one block $\underline{u}$ are fulfilled, the iterations can be stopped and the parameters $\hat{u}_\kappa$ can be estimated, e.g., using an MMSE estimator [3].

5 Suboptimal Decoding Without Channel State Information

In certain circumstances, no channel state information is available at the receiver. Then, MAP (or LogMAP) decoding fails and the suboptimal MaxLogMAP algorithm becomes an alternative [17]. However, the application of the suboptimal MaxLogMAP algorithm in the ISCD framework leads to performance losses of $\approx 0.7\,\mathrm{dB}$. The reason for these losses is that the MaxLogMAP decoder overestimates the extrinsic information at its output. A simple yet effective remedy against this overestimation is the "normalized MaxLogMAP" algorithm described for instance in [23]. After iteration i, the extrinsic output of the MaxLogMAP decoder is multiplied by an (iteration dependent) constant $\gamma(i)$ as indicated in Fig. 1. The additional computational complexity introduced by this multiplication is negligible compared to the overall complexity of the MaxLogMAP decoder and the SDSD. The $\gamma(i)$ are determined once in advance by measurements as described in [23]: At a channel quality where the performance of LogMAP and MaxLogMAP decoding differ the most, the factor $\gamma(i)$ is obtained using

$$\gamma(i) = \frac{\mathrm{E}\left\{L_{\text{CD,LogMAP}}^{[\text{ext}]}\right\}}{\mathrm{E}\left\{L_{\text{CD,MaxLogMAP}}^{[\text{ext}]}\right\}} \tag{12}$$

where $\mathrm{E}\{\cdot\}$ denotes expectation. We only evaluate (12) for the cases where $\mathrm{sign}\left\{L_{\text{CD,LogMAP}}^{[\text{ext}]}\right\} = \mathrm{sign}\left\{L_{\text{CD,MaxLogMAP}}^{[\text{ext}]}\right\}$ and $\left|L_{\text{CD,LogMAP}}^{[\text{ext}]}\right| < \left|L_{\text{CD,MaxLogMAP}}^{[\text{ext}]}\right|$. In a first step, the factor $\gamma(1)$ is obtained. Using this factor, the measurement can then be carried out for 2 iterations to obtain $\gamma(2)$ etc. For details, we refer the reader to [23].

6 Simulation Results

The capabilities of the proposed ISCD system with irregular index assignments scheme are demonstrated by a simulation example. The *parameter signal-to-noise ratio* (SNR) between the originally generated parameters u and the reconstructed estimated parameters $\hat{u}$ is used for quality evaluation. The parameter SNR is plotted for different values of E_u/N_0, with E_u denoting the energy per source parameter u_κ ($E_u = \bar{M}^* \cdot E_\mathrm{s}$). Additionally, the bit error probability P_b of the equivalent BSC channel is given on top of Fig. 3. Instead of using any specific speech, audio, or video encoder, we use the system setup already introduced in Section 3 with $K_S = 250$ statistically independent source parameters $\underline{u}$ modelled by K_S independent 1st order Gauss-Markov processes with auto-correlation $\rho = 0.9$. These auto-correlation values can be observed in typical speech and audio codecs, e.g., [24] for the scale factors in CELP codecs or MP3.

The simulation results are depicted in Fig. 3. A system with a non-redundant, natural binary index assignment, a rate $1/2$ convolutional code with memory $J = 3$, hard-decision Viterbi decoding and source decoding by table lookup serves as a reference.

While the system utilizing the regular index assignment BC_8^{16} achieves considerable gains compared to the reference, additional gains of $\approx 0.5\,\mathrm{dB}$ can be obtained by using the irregular index assignment from Section 3. If the *sphere packing bound* (SPB) is used to approximate the behavior for transmissions with a finite block length, the proposed system can reach the OPTA-SPB limit [19].

The number of utilized iterations is depicted in the lower part of Fig. 3. The maximum number of iterations for the system utilizing a regular index assignment is

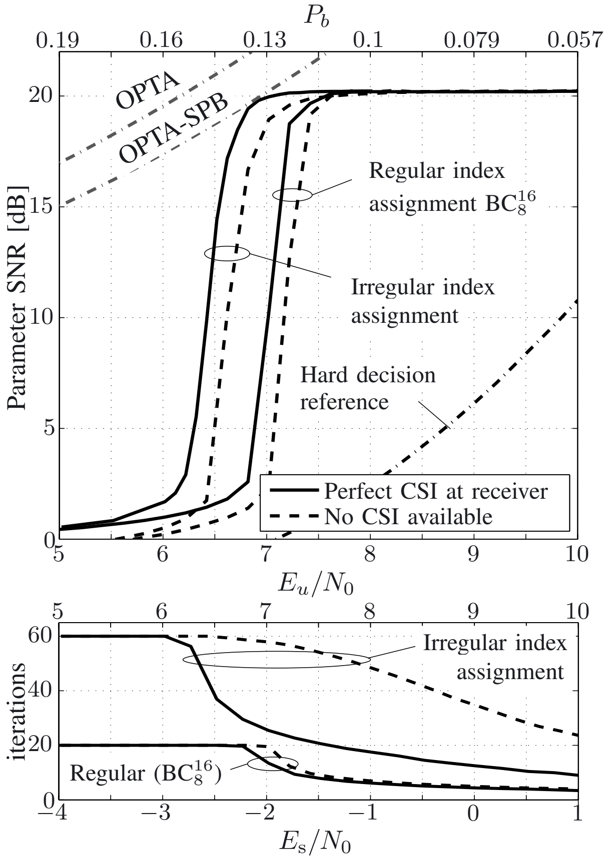

Fig. 3. Parameter SNR and mean number of iterations for a system with regular and irregular index assignments

fixed to 20 (as the EXIT chart and simulations show that only up to 20 iterations are beneficial) while the system using the irregular index assignment is allowed to exploit up to 60 iterations due to the narrow decoding tunnel (see EXIT chart in Fig. 2). The number of utilized iterations rapidly decreases in the waterfall region, however, the system utilizing *irregular index assignments* (IIA) needs more iterations in the whole range of channel conditions.

Figure 4 depicts the evaluation of the inverse normalization factors $1/\gamma(i)$ for the 20 first iterations in both systems. The factors have been determined for the channel quality $E_u/N_0 = 7.3\,\mathrm{dB}$ in the case of the regular index assignment BC_8^{16} and for $E_u/N_0 = 6.9\,\mathrm{dB}$ in the case of the irregular index assignment given in Section 3. Instead of converging to $\gamma(i) = 1$ for large i as in [23], the $\gamma(i)$ converge to a value of about 0.65, i.e., the extrinsic information is continuously overestimated by the MaxLogMAP decoder.

If no CSI is available, the correction factors $\check{L}_c$ in (4) are determined by using the channel qualities which have been utilized to determine the normalization factors $\gamma(i)$ (as described above and in Section 5). First the bit error probability of the channel quality is determined using (2), then the correction factor $\check{L}_c$ can be determined using

$$\check{L}_c = \log_\mathrm{e}\left(\frac{1-P_b}{P_b}\right). \tag{13}$$

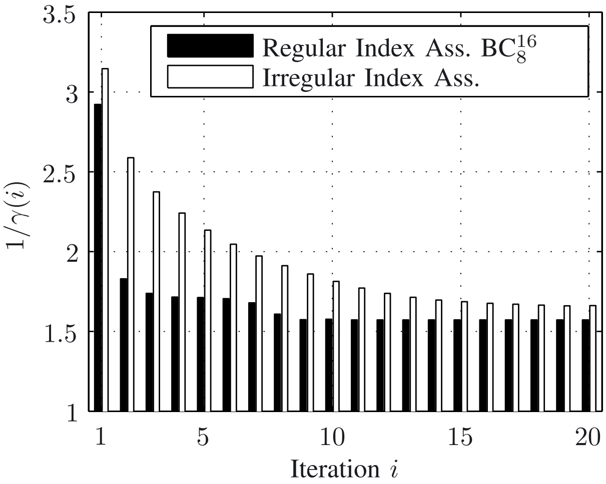

Fig. 4. Normalization factors $\gamma(i)$ for regular and irregular index assignments

Therefore, we get $\check{L}_c = 2.13$ for the system utilizing the regular index assignment BC_8^{16} and $\check{L}_c = 2.02$ for the system employing the irregular index assignment. The number of iterations needed if no CSI is available is generally higher than for the system without CSI, especially in the case of the irregular index assignment, as can be seen in Fig. 3. This could be explained by the fact that the single iterations only achieve little improvement in terms of mutual information and thus many iterations are needed to iterate through the decoding tunnel.

7 Conclusions

In our contribution, we presented the concept of *irregular index assignments* (IIA). Starting with a low-rate systematic generator matrix, the irregular index assignments are generated by multiplying the natural binary representation of the quantizer codebook indices with the first M^* rows of the generator matrix. Using the EXIT chart optimization algorithm known from the technique of irregular codes [7], the *iterative source-channel decoding* (ISCD) system can be optimized to yield near optimum performance. The generation of irregular index assignments using generator matrices yields a simple stopping criterion evaluated at the receiver. The iterative decoder can compute the parity check equations of the different parameters and if all parity check equations are fulfilled, decoding can be stopped. Furthermore, we have shown that in cases where no channel state information is available at the receiver, almost the same performance can be obtained if appropriate measures are taken, at the cost of an increased number of iterations, however.

Acknowledgment

This work was supported by the European Union under Grant FP6-2002-IST-C 020023-2 FlexCode.

References

[1] M. Adrat, P. Vary, and J. Spittka, “Iterative Source-Channel Decoder Using Extrinsic Information from Softbit-Source Decoding,” in *IEEE ICASSP*, Salt Lake City, USA, May 2001.

[2] N. Görtz, “On the Iterative Approximation of Optimal Joint Source-Channel Decoding,” *IEEE J. Select. Areas Commun.*, vol. 19, no. 9, pp. 1662–1670, Sept. 2001.

[3] T. Fingscheidt and P. Vary, “Softbit Speech Decoding: A New Approach to Error Concealment,” *IEEE Trans. Speech Audio Processing*, vol. 9, no. 3, pp. 240–251, Mar. 2001.

[4] P. Vary and R. Martin, *Digital Speech Transmission - Enhancement, Coding and Error Concealment*. John Wiley & Sons, Ltd., 2006.

[5] A. Ashikhmin, G. Kramer, and S. ten Brink, “Extrinsic Information Transfer Functions: Model and Erasure Channel Properties,” *IEEE Trans. Inform. Theory*, Nov. 2004.

[6] M. Tüchler, “Design of Serially Concatenated Systems Depending on the Block Length,” *IEEE Trans. Commun.*, vol. 52, no. 2, pp. 209–218, Feb. 2004.

[7] M. Tüchler and J. Hagenauer, “EXIT Charts of Irregular Codes,” in *Proc. of Conf. on Information Sciences and Systems (CISS)*, Princeton University, Mar. 2002.

[8] S. ten Brink, “Convergence Behaviour of Iteratively Decoded Parallel Concatenated Codes,” *IEEE Trans. Commun.*, vol. 49, no. 10, pp. 1727–1737, Oct. 2001.

[9] M. Adrat, P. Vary, and T. Clevorn, “Optimized Bit Rate Allocation for Iterative Source-Channel Decoding and its Extension towards Multi-Mode Transmission,” in *14th IST Mob. & Wirel. Comm. Summit*, Dresden, June 2005.

[10] T. Clevorn, M. Adrat, and P. Vary, “Turbo DeCodulation using Highly Redundant Index Assignments and Multi-Dimensional Mappings,” in *Intl. Symposium on Turbo Codes & Related Topics*, Munich, Apr. 2006.

[11] T. Clevorn, P. Vary, and M. Adrat, “Iterative Source-Channel Decoding using Short Block Codes,” in *IEEE ICASSP*, Toulouse, France, May 2006.

[12] S. Heinen and P. Vary, “Source-Optimized Channel Coding for Digital Transmission Channels,” *IEEE Trans. Commun.*, vol. 53, no. 4, pp. 592–600, Apr. 2005.

[13] T. Clevorn, J. Brauers, M. Adrat, and P. Vary, “Turbo DeCodulation: Iterative Combined Demodulation and Source-Channel Decoding,” *IEEE Commun. Lett.*, vol. 9, no. 9, Sept. 2005.

[14] L. Schmalen, T. Clevorn, and P. Vary, “Iterative Source-Coded Equalization: Turbo Error Concealment for ISI Channels,” in *Proc. of IEEE SPAWC*, Helsinki, June 2007.

[15] J. Hagenauer, E. Offer, and L. Papke, “Iterative Decoding of Binary Block and Convolutional Codes,” *IEEE Trans. Inform. Theory*, vol. 42, no. 2, pp. 429–445, Mar. 1996.

[16] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal Decoding of Linear Codes for Minimizing Symol Error Rate,” *IEEE Trans. Inform. Theory*, vol. 20, pp. 284–287, Mar. 1974.

[17] P. Robertson, E. Villebrun, and P. Hoeher, “A Comparison of Optimal and Sub-Optimal MAP Decoding Algorithms Operating in the Log Domain,” in *Proc. IEEE Int. Conf. Commun. (ICC)*, Seattle, USA, 1995, pp. 1009–1013.

[18] M. Adrat and P. Vary, “Iterative Source-Channel Decoding: Improved System Design Using EXIT Charts,” *EURASIP Jour. Appl. Sig. Proc.*, vol. 205, no. 6, pp. 928–941, May 2005.

[19] T. Clevorn, L. Schmalen, P. Vary, and M. Adrat, “On The Optimum Performance Theoretically Attainable for Scalarly Quantized Correlated Sources,” in *Proc. of ISITA*, Seoul, Korea, Nov. 2006.

[20] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. Chichester: J. Wiley & Sons, 2006.

[21] M. Adrat, T. Clevorn, J. Brauers, and P. Vary, “Minimum Terms of Residual Redundancy for Successful Iterative Source-Channel Decoding,” *IEEE Communications Letters*, vol. 10, no. 11, pp. 778 – 780, Nov. 2006.

[22] R. G. Gallager, *Low-Density Parity-Check Codes*. Cambridge, MA, USA: M.I.T. Press, 1963.

[23] J. Chen and M. P. C. Fossorier, “Near Optimum Universal Belief Propagation Based Decoding of Low-Density Parity Check Codes,” *IEEE Trans. Commun.*, vol. 50, no. 3, Mar. 2002.

[24] R. Thobaben, “Iterative Quellen- und Kanaldecodierung für Codes variabler Länge,” Ph.D. dissertation, Christian-Albrechts-Universitt zu Kiel, 2007, (in German).