Iterative Source-Channel Decoding: Design of Redundant Index Assignments

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Abstract

The exploitation of the residual redundancy of source codec parameters by using the Turbo principle is known as iterative source-channel decoding (ISCD). The concept of redundant index assignments was proposed as a powerful improvement for ISCD. In this contribution design and optimization guidelines for such redundant index assignments are presented. Commonly, the redundant index assignment is based on a simple block code. Using extrinsic information transfer (EXIT) chart we demonstrate that weak block codes such as a repetition code should be preferred. Simulation results show significant performance improvements with redundant index assignments based on this guideline.

1 Introduction

With the discovery of Turbo codes channel decoding close to the Shannon limit has become possible with moderate computational complexity. In the past years the Turbo principle has been extended to other receiver components. One of these extensions is *iterative source-channel decoding* (ISCD), e.g., [1, 2, 3], which allows to exploit the residual redundancy in source codec parameters such as scale factors or predictor coefficients for speech, audio, and video signals in a Turbo process. This residual redundancy occurs due to imperfect source encoding resulting, e.g., from delay constraints. The *a priori* knowledge on the residual redundancy, e.g., non-uniform probability distribution or auto-correlation, is utilized for *error concealment* by a derivative of a *soft decision source decoder* (SDSD) [4, 5], which exchanges *extrinsic* reliabilities with a channel decoder.

A key element in the optimization of an ISCD scheme is the index assignment (IA) in the source encoder from quantizer levels to bit patterns [6]. In [7] the concept of redundant index assignments is presented, which gives a significant performance improvement and enables a multi-mode ISCD system. More results on redundant index assignments can be found in [8, 9]. In this letter we analyze the redundant index assignments and optimization criteria in [7, 8, 9], propose a novel optimization concept, and demonstrate by simulation that the new simple redundant index assignments outperform the previously known ones. The new redundant index assignments resemble a simple repetition code in the given example. Some similarities to repeat-accumulate codes [10] can be observed.

2 The ISCD System Model

2.1 Conventional ISCD

In Fig. 1 the baseband model of iterative source-channel decoding is depicted. At time instant τ, a source encoder determines a frame $\underline{u}_\tau$ consisting of K_S source codec parameters $u_{\kappa,\tau}$ with $\kappa = 1,\dots K_S$ denoting the position in the frame. The single elements $u_{\kappa,\tau}$ of $\underline{u}_\tau$ are assumed to be statistically independent from each other. By scalar quantization each value $u_{\kappa,\tau}$ is individually mapped to a quantizer reproduction level $\bar{u}_\kappa^{(\xi_\kappa)}$ with index $\xi_\kappa = 0,\dots Q-1$. The number of quantizer levels is usually assumed as $Q = 2^M$. To each quantizer level $\bar{u}_\kappa^{(\xi_\kappa)}$ selected at time instant τ a unique bit pattern $\mathbf{x}_{\kappa,\tau}$ of M bits is assigned according to the index assignment Γ, $\mathbf{x}_{\kappa,\tau} = \Gamma(\xi_\kappa)$. The frame of K_S bit patterns is denoted by $\underline{\mathbf{x}}_\tau$. The bit interleaver π scrambles the incoming frame $\underline{\mathbf{x}}_\tau$ of data bits to $\underline{\tilde{\mathbf{x}}}_\tau$ in a deterministic manner. We restrict the interleaving to a single time frame with index τ and omit the time frame index τ in the following where appropriate.

For channel encoding $\mathcal{C}$ of $\underline{\tilde{\mathbf{x}}}$ we can use, e.g., a standard terminated *recursive non-systematic convolutional* (RNSC) code of constraint length $J+1$ and rate $r_\mathcal{C}$. The resulting codeword is mapped to bipolar bits $\ddot{y} \in \{\pm 1\}$, e.g., for BPSK transmission with symbol energy $E_s = 1$. On the channel additive white Gaussian noise (AWGN) n with a known power spectral density of $\sigma_n^2 = N_0/2$ is applied, i.e., $\ddot{z} = \ddot{y} + n$.

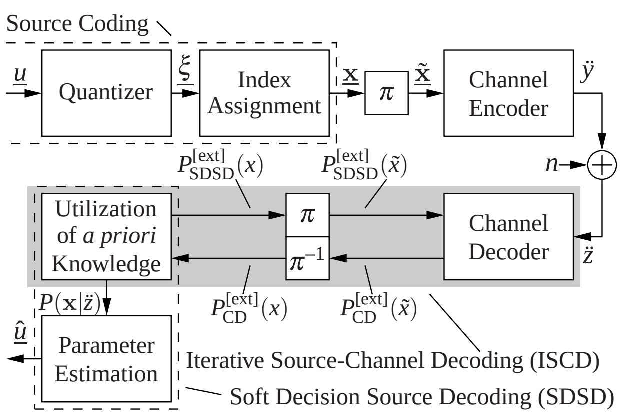

Figure 1: Baseband model of ISCD

The received symbols $\ddot{z}$ are evaluated in a Turbo process, which exchanges *extrinsic* reliabilities between the channel decoder (CD) and the *soft decision source decoder* (SDSD). Such reliability information can either be evaluated in terms of probabilities $P(\cdot)$ or as *log-likelihood* ratios (L-values) $L(\cdot)$.

The ISCD receiver is described in detail, e.g., in [1, 2, 5, 6]. For convolutional codes the equations for the *extrinsic* probabilities are well known. The SDSD determines *extrinsic* information mainly from the natural residual source redundancy which remains in the bit patterns $\mathbf{x}_{\kappa,\tau}$ after source encoding. Such residual redundancy appears on *parameter-level*, e.g., in terms of a non-uniform distribution $P(\mathbf{x}_{\kappa,\tau})$, in terms of correlation, or in any other possible mutual dependency in time τ. The algorithm how to compute the *extrinsic* $P_{\text{SDSD}}^{[\text{ext}]}(x)$ has been detailed, e.g., in [1, 2, 5, 6].

2.2 ISCD with Redundant Index Assignments

Redundant index assignments for ISCD were proposed in [7]. In comparison to a conventional ISCD scheme with channel coding of rate r_{C}, the number of bits assigned to each quantizer level $\bar{u}_{\kappa}^{(\xi_\kappa)}$ can be increased by a factor of up to $1/r_{\text{C}}$, assuming a constant gross bit rate by reduction of the channel coding rate at the same time. For example, if $r_{\text{C}} = 1/2$ channel coding and $M = 3$ bits per Q quantizer level were used for conventional ISCD, with redundant index assignments and $r_{\text{C}} = 1$ bit patterns $\mathbf{x}$ of size $M^\star = M/r_{\text{C}} = 6$ can be assigned to each quantizer level.

The obvious choice now would be to use $Q^\star = 2^{M^\star} = 64$ quantizer levels to reduce the quantization noise. However, the only constraint for $Q^\star$ is $Q^\star \leq 2^{M^\star}$. Thus, if, e.g., the quantization noise of $Q^\star = 8$ quantization levels is tolerable (as in the conventional case with $M = 3$ bits), we can apply a redundant index assignment with $M^\star = 6$ bits for each level. The benefits of the latter approach with $Q^\star < 2^{M^\star}$ over the obvious choice $Q^\star = 2^{M^\star}$ in case of channel errors have been analyzed in detail in [7]. Such an index assignment with $Q^\star < 2^{M^\star}$ can be considered as a (potentially non-linear) block code with the binary representation of the quantizer level index $\{\xi\}_2$, $\xi = 0, \ldots Q^\star - 1$, as input (see also [11, 12] for non-iterative schemes). In case of an index assignment by a linear block code with generator matrix $\mathbf{G}^\Gamma$ we get [8]

$$\mathbf{x} = \Gamma(\xi) = (\{\xi\}_2) \cdot \mathbf{G}^\Gamma \quad . \tag{1}$$

In [8] simple block codes (BC) were chosen with maximal minimum Hamming distance $d_{\min}$. For the exemplary $Q^\star = 8$ and $M^\star = 6$ we get for instance

$$\mathbf{G}_{\text{BC}(6,3)}^\Gamma = \begin{pmatrix} 1\,0\,0\,1\,1\,0 \\ 0\,1\,0\,1\,0\,1 \\ 0\,0\,1\,0\,1\,1 \end{pmatrix} \quad , \tag{2}$$

with $d_{\min}(\mathbf{G}_{\text{BC}(6,3)}^\Gamma) = 3$. The corresponding index assignment is denoted by $\Gamma \hat{=} \text{BC}_{M^\star=6}^{Q^\star=8}$. Thus, e.g., for $\xi = 3$ we get with (1)

$$\mathbf{x} = \Gamma(3) = (\{3\}_2) \cdot \mathbf{G}^\Gamma = (011) \cdot \mathbf{G}^\Gamma = 011110 \quad . \tag{3}$$

3 Optimization of Redundant Index Assignments

The advantage of redundant index assignments with respect to conventional index assignments can be best observed in an EXIT chart [6]. In Fig. 2 the EXIT charts of ISCD with a conventional index assignment and ISCD with a redundant index assignment are compared. For the simulation settings we refer to Chapter 4. The index assignments are identified by $(\cdot)_M^Q$ or $(\cdot)_{M^\star}^{Q^\star}$. NB stands for *natural binary*, EO for *EXIT optimized* [6], and BC for *block coded* [8]. In the left EXIT chart we can observe that the EXIT characteristics T of conventional index assignments are limited to a maximum extrinsic mutual information $I_{\text{SDSD}}^{[\text{ext,max}]} \ll 1$ bit [6]. As the stopping intersection with the channel code is usually close to $I_{\text{SDSD}}^{[\text{apri}]} = 1$ bit, the maximization of $I_{\text{SDSD}}^{[\text{ext,max}]}$ is a very good optimization criterion [6].

In contrast, the EXIT characteristics of redundant index assignments can reach the point (1,1) in the EXIT chart, if the minimum Hamming distance $d_{\min}$ between the bit patterns $\mathbf{x}$ of the index assignment is larger than 1, i.e., $d_{\min} \geq 2$ [13, 8, 14, 9]. Note, for unequally distributed bits x actually only the point $(H(x), H(x))$ can be reached, with $H(x)$ being the entropy of x. Both EXIT characteristics $T(\mathrm{C}_{\text{RNSC}}^{r_{\text{C}}})$ of the channel codes reach (1,1), but due to the different r_{C}, they differ significantly.

While in the left plot with the conventional index assignment the point (1,1) can never be approached by the decoding trajectory at all due to $I_{\text{SDSD}}^{[\text{ext,max}]} < 1$, the point (1,1) can be reached in the right plot with the redundant index assignment as soon as the tunnel opens noticeably, which is the case for a E_s/N_0 slightly above -4 dB. Thus, the critical region is now at a relatively low $I_{\text{SDSD}}^{[\text{apri}]}$, around $I_{\text{SDSD}}^{[\text{apri}]} \approx 0.2$ bit, where the tunnel opens, because any index assignment with $d_{\min} \geq 2$ guarantees reaching (1,1).

The block coded index assignments in [8] were chosen for a high $d_{\min}$, e.g., $d_{\min}(\mathbf{G}_{\text{BC}(6,3)}^\Gamma) = 3$. The EXIT optimized index assignments of [9] (not to be confused with the EXIT optimized index assignments of [6]) result actually in a single parity-check code (SPC) for $\mathbf{G}^\Gamma$ ("Mapping 2" in [9], cmp. [13, 14] and $\mathbf{G}_{\text{BC}(6,5)}^\Gamma$ in [15]). Note, a SPC is the only possibility to achieve $d_{\min} \geq 2$ for $M^\star = M+1$ as used in [13, 14, 9], when the index assignment shall be systematic, i.e., $\{\xi\}_2$ is part of $\mathbf{x}$. The proof is obvious. If two $\{\xi\}_2$ differ by only one bit of the M bits, one has

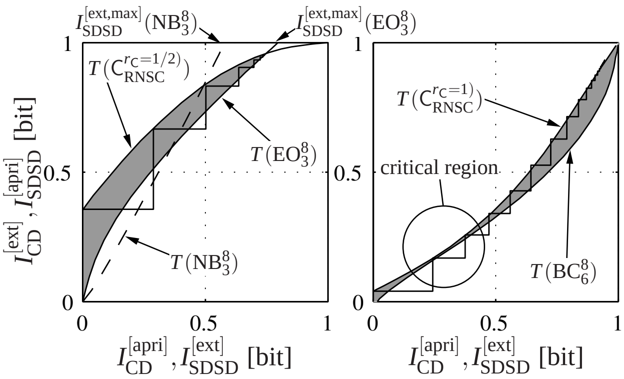

Figure 2: EXIT chart at $E_s/N_0 = -4$ dB for ISCD with conventional IA (left) and redundant IA (right).

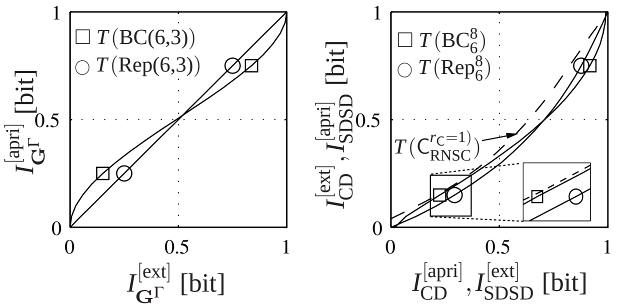

Figure 3: EXIT characteristics for block codes $\mathbf{G}^\Gamma$ (left) and EXIT chart at $E_s/N_0 = -4$ dB with corresponding redundant IAs (right).

an even Hamming weight, the other one an odd. If the two corresponding $\mathbf{x}$ with $M^\star = M+1$ bit shall have $d_{\min} \geq 2$, the added bit must be different for the two $\mathbf{x}$. Afterwards, both $\mathbf{x}$ have an even weight or both have an odd weight. As this holds for all combinations of $\{\xi\}_2$, this results in an even or odd SPC for $\mathbf{G}^\Gamma$.

3.1 Novel Repetition Coded Index Assignments

If a redundant index assignment is linear, i.e., (1) can be used, it can be considered as a combination of a conventional natural binary index assignment, e.g., NB_3^8 in Fig. 2, and a block code with a generator matrix $\mathbf{G}^\Gamma$. Although the EXIT characteristic T of the final redundant index assignment cannot be directly computed from the EXIT characteristics $T(\mathrm{NB}_M^Q)$ and $T(\mathbf{G}^\Gamma)$ of the components, it is worth to take a a closer look at $T(\mathbf{G}^\Gamma)$. In Fig. 3 the $T(\mathbf{G}^\Gamma)$ of the BC(6,3) of (2) and of a repetition code,

$$\mathbf{G}_{\mathrm{Rep}(6,3)}^\Gamma = \begin{pmatrix} 1\,0\,0\,1\,0\,0 \\ 0\,1\,0\,0\,1\,0 \\ 0\,0\,1\,0\,0\,1 \end{pmatrix} \quad (\text{corresp. IA: } \Gamma \hat{=} \mathrm{Rep}_{M^\star=3}^{Q^\star=8}), \tag{4}$$

with $d_{\min}(\mathbf{G}_{\mathrm{Rep}(6,3)}^\Gamma) = 2$, are depicted in the left subplot. Note the swapped axes. It can be observed that $\mathbf{G}_{\mathrm{Rep}(6,3)}^\Gamma$ yields the highest $I_{\mathbf{G}^\Gamma}^{[\mathrm{ext}]}$ for $I_{\mathbf{G}^\Gamma}^{[\mathrm{apri}]} < 0.5$ bit and $\mathbf{G}_{\mathrm{BC}(6,3)}^\Gamma$ the lowest. For $I_{\mathbf{G}^\Gamma}^{[\mathrm{apri}]} > 0.5$ bit the order reverses.

A similar behavior can be observed in the right subplot of Fig. 3 for the corresponding redundant index assignments, BC_6^8 and Rep_6^8. For $I_{\mathrm{SDSD}}^{[\mathrm{apri}]} > 0.5$ bit all redundant index assignments provide a sufficient gap to $T(\mathrm{C}_{\mathrm{RNSC}}^{r_\mathrm{C}=1})$, as all index assignments fulfill $d_{\min} \geq 2$. For $I_{\mathrm{SDSD}}^{[\mathrm{apri}]} \approx 0.2$ bit the gap is almost closed for BC_6^8 at $E_s/N_0 = -4$ dB, while a sufficient tunnel still exists for Rep_6^8. This advantage of Rep_6^8 can be again found in the simulation results in Chapter 4.

To summarize, for redundant index assignments with $d_{\min} \geq 2$ the critical region of the EXIT characteristic is at low $I_{\mathrm{SDSD}}^{[\mathrm{apri}]}$, i.e., $I_{\mathrm{SDSD}}^{[\mathrm{apri}]} < 0.5$ bit, and not at high $I_{\mathrm{SDSD}}^{[\mathrm{apri}]}$. The EXIT characteristic of an underlying block code $\mathbf{G}^\Gamma$ can serve as indication of the EXIT characteristic of the redundant index assignment. Block codes $\mathbf{G}^\Gamma$ with a low $d_{\min}$ (but ≥ 2), such as a repetition code, provide the best performance in the critical region of the EXIT characteristic, and thus, also the best system performance.

4 Simulation Results

The capabilities of the repetition code index assignments shall be demonstrated by simulation. Similar settings as in, e.g., [5, 6, 8] are used. $K_S = 250$ statistically independent parameters per frame $\underline{u}_\tau$, modeled by independent Gauss-Markov processes with $\sigma_u^2 = 1$ and auto-correlation $\rho = 0.9$, are Lloyd-Max quantized to $Q = Q^\star = 8$ levels with $M = 3$ or $M^\star = 6$ bits/parameter. RNSC codes with the generator polynomials $\mathbf{G}_\mathrm{C}^{r_\mathrm{C}=1/2} = (\frac{1+D+D^3}{1+D+D^2+D^3}, \frac{1+D^2+D^3}{1+D+D^2+D^3})$ for $r_\mathrm{C} = 1/2$ and $\mathbf{G}_\mathrm{C}^{r_\mathrm{C}=1} = (\frac{1}{1+D+D^2+D^3})$ for $r_\mathrm{C} = 1$ are used for channel coding.

In the simulation results in Fig. 4 it can be observed that all ISCD schemes with 25 iterations outperform the best non-iterative scheme by several dB in E_s/N_0. Note, the depicted non-iterative scheme uses SDSD. Today's systems with hard output channel decoding perform even worse.

The light gray area marks the gain by the known redundant index assignment BC_6^8. The maximum parameter SNR can be retained to much lower E_s/N_0 than with an optimized conventional index assignment EO_3^8. The dark gray area marks the additional gain by the new repetition coded index assignment Rep_3^8. On the one hand, the gap to the theoretical limit is closed. The limit is the OPTA-SPB bound [16], the *optimum performance theoretically attainable* using the *sphere packing bound* to incorporate the finite block size. On the other hand, the performance in the waterfall region is also improved. The gap to ISCD with a conventional index assignment (EO_3^8) is almost closed from $\Delta_{E_s/N_0} \approx 0.5$ dB for BC_6^8.

The simulation results in Fig. 5 show that similar observation can be made for $Q = Q^\star = 16$ quantizer levels and $M = 4$ or $M^\star = 8$ bits/parameter. The BC_8^{16} index assignment is based on a BC(8,4) block code with generator matrix $\mathbf{G}_{\mathrm{BC}(8,4)}^\Gamma$ and $d_{\min}(\mathbf{G}_{\mathrm{BC}(8,4)}^\Gamma) = 4$. In contrast, the index assignment Rep_8^{16} of a Rep(8,4) repetition code has $d_{\min}(\mathbf{G}_{\mathrm{Rep}(8,4)}^\Gamma) = 2$. The improvement of Rep_8^{16} compared to BC_8^{16} is $\Delta_{E_s/N_0} \approx 0.8$ dB. The new repetition coded index assignment Rep_8^{16} outperforms the known index assignments, an optimized conventional index assignment such as EO_4^{16} as well as redundant index assignments such as BC_8^{16}, in the whole relevant E_s/N_0 range.

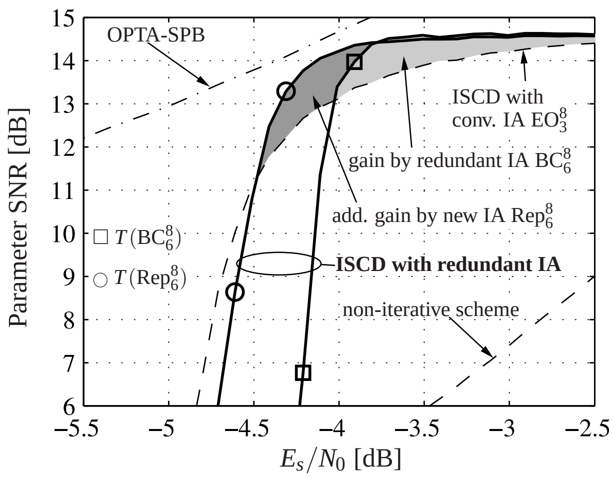

Figure 4: Parameter SNR simulation results with $Q = Q^\star = 8$ levels.

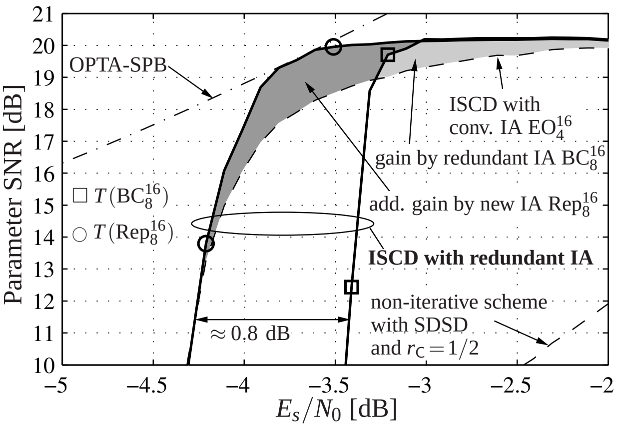

Figure 5: Parameter SNR simulation results with $Q = Q^{\star} = 16$ levels.

5 Conclusion

Using the EXIT chart tool we analyze redundant index assignments for ISCD and derive new design and optimization criteria. For an early convergence the extrinsic mutual information for low to medium a priori information should be maximized. Weak block codes such as simple repetition codes fulfill this guideline best. Their loss for medium to high a priori information has no relevant effect. Compared to known redundant and conventional index assignments, the simulation results demonstrate a significant performance improvement with a repetition coded redundant index assignment.

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