Modern Techniques for Flexible, Iterative Source-Channel Decoding

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Overview

- Soft Decision Source Decoding
- The Turbo Concept of Iterative Decoding
- Generation of Extrinsic Information for the Source Decoder
- Iterative Source-Channel Decoder
- Adaptive, Flexible, Multi-Mode Iterative Source-Channel Decoding scheme
Introduction

• Channel coding cannot prevent occurrence of residual bit errors in the case of adverse channel conditions leading to a severe degradation of the signal quality
• Annoying effects can be reduced or even eliminated by means of error concealment
• Real source coding schemes contain residual redundancy for reasons of delay, complexity and nonstationarity

• Shannon 1948:

„However, any redundancy in the source will usually help if it is utilized at the receiving point. [...] redundancy will help combat noise.“
Conventional Transmission

- Quantization & Index Assignment
- Interleaver
- Channel Encoder
- Interleaver
- Modulator / Mapper

Parameter Extraction

Source

$u$

$p(u)$

$u_1$

$u_K$

$t$

$t$
Conventional Transmission

- **Transmitter**
  - Quantization & Index Assignment
  - Interleaver
  - Channel Encoder
  - Interleaver
  - Modulator / Mapper

  \[
  \text{Parameter SNR} = \frac{E\{u^2\}}{E\{(u - \hat{u})^2\}}
  \]

- **Receiver**
  - Demodulator / Demapper
  - Channel Decoder
  - Parameter Lookup

  \[
  \hat{u}
  \]
Exploitation of residual redundancy for quality improvement

1D \textit{a priori} knowledge (parameter distribution)

\begin{align*}
\mathbf{u} &= (u_1, u_2, \ldots) \\
\mathbf{x} &= (x_1, x_2, \ldots) \\
\mathbf{y} &= \\
\mathbf{z} &= \\
\mathbf{c} &= (c_1, c_2, \ldots) \\
\hat{u} &= \\
\end{align*}

\textit{a posteriori} probabilities:

\begin{equation}
P(\hat{u} | \mathbf{c}) = K \cdot P(c | \mathbf{x}) \cdot P(\hat{u})
\end{equation}

can be calculated using channel statistics

normalization constant

[Fingscheidt01]
Soft Decision Source Decoding

Exploitation of residual redundancy for quality improvement

1D *a priori* knowledge (parameter distribution)

\[ u = (u_1, u_2, \ldots) \]

\[ x = (x_1, x_2, \ldots) \]

\[ y \]

\[ z \]

\[ c = (c_1, c_2, \ldots) \]

\[ \hat{u} \]

\[ P(\hat{u}|c) = K \cdot P(c|\hat{u}) \cdot P(\hat{u}) \]

* a posteriori * probabilities: \( P(\hat{u}|c) \)

Normalization constant can be calculated using channel statistics

Fingscheidt01
Soft Decision Source Decoding

Exploitation of residual redundancy for quality improvement

Parameter estimation instead of table lookup
(MMSE estimation using \textit{a posteriori} probabilities)

\[
\hat{\mathbf{u}} = \sum_{i=1}^{Q} \tilde{\mathbf{u}}^{(i)} \cdot P(\tilde{\mathbf{u}}^{(i)} | \mathbf{c})
\]
Exploitation of residual redundancy for quality improvement

1D *a priori* knowledge (parameter distribution)

2D *a priori* knowledge (parameter correlation)

*a posteriori* probabilities can be calculated by the recursion:

\[
P(\bar{u}_t|c) = K \cdot P(c_t|x_t) \cdot \sum_{i=1}^{Q} P(x_t|x_{t-1}) \cdot P(\bar{u}_{t-1}|c)
\]

channel statistics

[Fingscheidt01]
Soft Decision Source Decoding

Exploitation of residual redundancy for quality improvement

1D \textit{a priori} knowledge (parameter distribution)

2D \textit{a priori} knowledge (parameter correlation)

A \textit{posteriori} probabilities can be calculated by the recursion:

\[
P(\bar{u}_t | \bar{c}) = K \cdot P(c_t | \bar{u}_t) \cdot \sum_{i=1}^{Q} P(\bar{u}_t | \bar{u}_{t-1}) \cdot P(\bar{u}_{t-1} | \bar{c})
\]

[Fingscheidt01]
Soft Decision Source Decoding

Exploitation of residual redundancy for quality improvement

1D \textit{a priori} knowledge (parameter distribution)

2D \textit{a priori} knowledge (parameter correlation)

Parameter estimation instead of table lookup
(MMSE estimation using \textit{a posteriori} probabilities)

\[
\hat{u}_t = \sum_{i=1}^{Q} \bar{u}_t^{(i)} \cdot P(\bar{u}_t^{(i)} | C)
\]

[Fingscheidt01]
Iterative Channel Coding

• Timeline of iterative decoding techniques
  – LDPC codes introduced in 1963 [Gallager63] but forgotten due to the relatively high complexity at that time
  – Turbo codes invented in 1993 [Berrou93]. Allow near-Shannon limit decoding with moderate complexity
  – LDPC codes rediscovered in 1998 [MacKay98]. Decoding is also performed iteratively using belief propagation
  – Extension of the iterative decoding to other receiver components, e.g.
    • equalization (Turbo Equalization) [Douillard95]
    • modulation (BICM-ID) [Xi98]
    • multi-user detection (Turbo-MUD) [Alexander98]
    • source decoding (ISCD) [Adrat01], [Goertz01], [Guyader01]
Turbo Codes, Concept

Concatenated Encoding

Iterative Turbo Decoding

advantages by iterative feedback!

[Berrou93, Benedetto98]
Extrinsic Information

“Information from neighboring bit positions”

- Example: Parity Check Code

\[ y_P = y_1 \oplus y_2 \]

- Extrinsic information:
  - transmitter \( y_1 = y_2 \oplus y_P \)
  - receiver \( \tilde{z}_{e1} = z_2 \oplus z_P \)
Extrinsic Information from SDSD

- Generation of extrinsic information by the soft decision source decoder [Adrat03]

Information from channel decoder:
2 rightmost bits are 01
Extrinsic Information from SDSD

• Generation of extrinsic information by the soft decision source decoder [Adrat03]

Information from channel decoder:
2 rightmost bits are 01
Extrinsic Information from SDSD

- Generation of extrinsic information by the soft decision source decoder [Adrat03]

Information from channel decoder:
2 rightmost bits are 01

This information can be fed back to the channel decoder to improve channel decoding!

\[ P(x_1 = 0 | (x_2, x_3) = (0, 1)) < P(x_1 = 1 | (x_2, x_3) = (0, 1)) \]
Extrinsic Information from SDSD

- Generation of extrinsic information by the soft decision source decoder [Adrat03]

Usually, the bits are not perfectly known!

→ marginalization and application of Bayes theorem

$$P(x_1 = 1 | c_{\backslash 1}) = C \cdot \sum_{x_{\backslash 1}} P(x_{\backslash 1}, x_1 = 1) \cdot P(c_{\backslash 1} | x_{\backslash 1})$$

Bit pattern $X$

$x_1$ $X_{\backslash 1}$

channel transition probabilities

a priori knowledge

000 001 010 011 100 101 110 111
Extrinsic Information from SDSD

- Generation of extrinsic information by the soft decision source decoder \([\text{Adrat03}]\)
  
  Usually, the bits are not perfectly known!

  → marginalization and application of Bayes theorem

\[
P(x_1 = 1 | c_\backslash_1) = C \cdot \sum_{x_\backslash_1} P(x_\backslash_1, x_1 = 1) \cdot P(c_\backslash_1 | x_\backslash_1)
\]

- Bit pattern \(X\)
  - a priori knowledge
  - channel transition probabilities
• Correlation of the source is exploited by the SDSD

![Diagram showing $p(u_t)$ and $p(u_t|u_{t-1})$]

• The quantization of the source modelled as Markov process can be represented using a trellis diagram
• Trellis representation of the source

\[ x_{t-1} = (0, 0, 0) \]
\[ x_{t-1} = (0, 0, 1) \]
\[ x_{t-1} = (1, 1, 0) \]
\[ x_{t-1} = (1, 1, 1) \]

\[ x_t = (0, 0, 0) \]
\[ x_t = (0, 0, 1) \]
\[ x_t = (1, 1, 0) \]
\[ x_t = (1, 1, 1) \]

States correspond to quantizer reproduction levels
State transitions correspond to conditional probabilities

Decoding using the BCJR (MAP) algorithm
[Bahl et al. 74], [Heinen 00], [Adrat 05]

• The quantization of the source modelled as Markov process can be represented using a trellis diagram
Extrinsic Information from a second decoder”

Extrinsic from 2nd decoder → Turbo component 1 → Extrinsic for 2nd decoder

A priori knowledge → Turbo component 1

Soft-information from channel → Turbo component 1 → A posteriori
Iterative Exchange of Extrinsic Information

- a posteriori decision after several iterations
Iterative Decoder Concepts

• Bit-Interleaved Coded Modulation & Iterative Decoding [Xi98]

• Iterative Source-Channel Decoding [Adrat01]

• Turbo DeCoderul [Clevorn05]
Iterative Source-Channel Decoding

- Design constraint: Parameter SNR e.g. $P^{[\text{ref}]} > 13$ dB

![Graph showing iterative decoding performance](image)

**AWGN / BPSK Modulation**
- 250 parameters/frame
- auto-correlation $\rho = 0.9$
- 3 Bit Lloyd-Max quantizer
- $r_c = 1/2$ convolutional code with 8 trellis states

**SDSD:**
- first order Markov model

**ISCD:**
- recursive non-systematic convolutional code
- EXIT optimized index assignment
- 10 iterations
Audio Examples

- Audio examples
  - improvement by iterative decoding

<table>
<thead>
<tr>
<th></th>
<th>Speech</th>
<th>Music</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 iteration</td>
<td><img src="audio1.png" alt="Audio" /></td>
<td><img src="audio1.png" alt="Audio" /></td>
</tr>
<tr>
<td>2 iterations</td>
<td><img src="audio2.png" alt="Audio" /></td>
<td><img src="audio2.png" alt="Audio" /></td>
</tr>
<tr>
<td>3 iterations</td>
<td><img src="audio3.png" alt="Audio" /></td>
<td><img src="audio3.png" alt="Audio" /></td>
</tr>
<tr>
<td>4 iterations</td>
<td><img src="audio4.png" alt="Audio" /></td>
<td><img src="audio4.png" alt="Audio" /></td>
</tr>
</tbody>
</table>

A-law PCM with 8-bit quantization

AWGN / BPSK Modulation
- 44.1 kHz sampling rate
- 300 samples/frame
- $r_C = 1$ convolutional code with 8 trellis states
- redundant $r_{IA} = \frac{1}{2}$ block coded index assignment
- $E_s/N_0 = 1$ dB (BER = 5.5%)

SDSD:
- exploiting unequal parameter distribution only (zeroth order apriori knowledge)

→ further improvement by exploiting correlation
Redundant Index Assignments

- Highly Redundant Index Assignments [Adrat05]

\[ Q = 8 \]
\[ w = 3 \]
\[ c = 6 \]

\[ r_C = w/c = 1/2 \]
\[ r_C = w^*/c = 1 \]
Iterative Source-Channel Decoding

- Extension towards a multi-mode system by variable source encoding ($Q$ quantizer levels)
- Fixed channel coding with rate $r = 1$

AWGN channel
- BPSK Modulation
- 250 parameters/frame
- auto-correlation $\rho = 0.9$
- Lloyd-Max quantizer with $Q$ levels
- 6 bits per parameter
- convolutional code with 8 trellis states
- redundant index assignment

![Graph showing parameter SNR vs. $E_s/N_0$ (dB)]

- Optimized ISCD ($r_C=1$) block coded IA, 25 iter.
- $Q = 8$ quantizer levels
- Theoretical Limit
- Classic ISCD ($r_C=1/2$) optimized IA, 10 iter.
Iterative Source-Channel Decoding

- Extension towards a multi-mode system by variable source encoding (Q quantizer levels)
- Fixed channel coding with rate \( r = 1 \)

**Plot Description**

- **AWGN channel**
  - BPSK Modulation
  - 250 parameters/frame
  - auto-correlation \( \rho = 0.9 \)
  - Lloyd-Max quantizer with \( Q \) levels
  - 6 bits per parameter
  - convolutional code with 8 trellis states
  - redundant index assignment

**Graph Details**

- **Optimized ISCD \((r_c=1)\)**
  - block coded IA, 25 iter.
  - \( Q = 8 \) quantizer levels

- **Theoretical Limit**

- **Classic ISCD \((r_c=1/2)\)**
  - optimized IA, 10 iter.

**Axes**

- \( E_s/N_0 [\text{dB}] \)
- Parameter SNR \( \mathcal{P} [\text{dB}] \)
Iterative Source-Channel Decoding

- Extension towards a multi-mode system by variable source encoding (Q quantizer levels)
- Fixed channel coding with rate $r = 1$

AWGN channel
- BPSK Modulation
- 250 parameters/frame
- auto-correlation $\rho = 0.9$
- Lloyd-Max quantizer with Q levels
- 6 bits per parameter
- convolutional code with 8 trellis states
- redundant index assignment
Conclusions

• Exploitation of residual source redundancy for soft decision source decoding
• Determination of extrinsic information by using residual source redundancy
• Iterative decoding concept extended to the source decoding step
  – Near capacity decoding of quantized, correlated sources
  – Adaptivity/Flexibility by using a multi-mode transmission scheme
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